Computational economics
(Qualitative economics aided by computational intelligence)*

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Abstract

In economics there are several complex learning themes and tasks connected with them difficult for deeper understanding of the learning subject. These are the reasons originating serious learning problems for students in the form of Virtual University because deeper understanding requires high level mathematical skills. The hope is that products of computational intelligence may help them solve such problems. Naturally, the meant complex economic problems and tasks have discrete, qualitative and nonlinear (noninvertible) nature resulting in increased level of difficulties. So with the term used in this paper one has to understand narrowly: “qualitative nonlinear computational economics”. For better understanding the very nature of the problem we are using as appropriate example actual simulation of the model of new ICT products monopolies in virtual laboratory built in the routines setting dominantly in the software iDmc.

1. Introduction

Qualitative computational economics is one of the newer parts of economics. From the view of very contents its subjects are those economic problems that along to complex economic systems, for example cyclical growth, monopolist’ searching, adaptively learning duopoly, multiplier-accelerator dynamics based on the different behaviour of investor (fundamentalists and chartists), etc. On the other hand, from the point of methods and analytical tools this part of economics are using results of progress in qualitative mathematics originated by Henri Poincaré, and for preliminary simple models solving are using PC-simulations, and experimentations in virtual laboratories built in appropriate software’s, for examples in MATLAB-SIMULINK, STELLA, VENSIM, SWARM, GEMODEL, Mathematica/Wolfram, and other too, including EXCEL spreadsheet processor. The most important feature for discerning this part of economics is the set of qualitative shapes emerging in discrete dynamic systems when they are undergoing iterations (recursion) and/or experimentation with parameters and initial coordinates of variables. Among such shapes there are: - nodes, - focuses, - saddles, - centres, - one parametric deterministic chaos, - limit cycles, Hopf-Andronov, and/or Neimark-Sacker bifurcations, saddle-node bifurcations, - saddle-focus bifurcations, - homoclinical and/or heteroclinical tangles, - knots, etc.; and they may be portrayed by: - time steps trajectories in evolving time; - state and/or phase trajectories in \( \mathbb{R}^2 \) of two variables; - cobweb portraits; - one control parameter bifurcation with first and/or with second variables; - two control parameters bifurcation in \( \mathbb{R}^2 \) (attractive basin of double controls); - cycles; - basin of attraction of two variables; - one Lyapunov’s exponent against some of control parameters; - Lyapunov’s exponents with two control parameters in \( \mathbb{R}^2 \); - absorbing area with possibility to create critical curves and/or attractors.

The students in various fields of economic subjects’ education in university level must face up to wide agenda of complex phenomena in learning process in bachelor and/or master level too. In the more difficult situations are finding ourselves students in distance form and in virtual university. Deep understanding of upon indicates complex learning matters are based on high level mathematical skill of students and on their ability to use appropriate tools. But such requirements aren’t fulfilled in every case. So the efficiency and quality of understanding and mastering meant subjects isn’t assured. In such situation it is at hand very simple and in many events suitable solution: that is experimentation in virtual laboratories. On the other hand, those methods aren’t panaceae. But, at least, can successfully help them better understand usage of abstract methods of cogitation and in deeper penetration to using efficient mathematical tools in complex system analysis. Based on many years experiences we come to the end that from the methodical and didactical point of view there are two dominant stage of progress achieved by using virtual laboratories in education process: the first is possibility for the student follow dialogue with complex problems mastered as a model in virtual laboratory by teacher, that is the students doing approval of his imaginations by experimentation, the second one and more efficient is building own authentic model by student and afterwards doing experiments with in. These two kinship and collaborator approaches are constructive in character unlike traditional approaches which are instructive.

The instigate purpose and very goals of author is grasping opportunity supplied with this conference and take off spread among wider community his and his fellow colleagues opinions and practical experiences with using

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virtual laboratories for better and deeper understanding complex phenomena in various learning economic disciplines. In actions with those problems they are using phenomenological and/or entirely qualitative approaches to meant subjects for elaborating preparatory basis needed for creation mathematical and/or simulation models of authentic learning subject in economics and other partial branches of economic science. In this paper the author will demonstrate only a few explicit examples of virtual laboratories, experimenting in them and shows graphical snapshots of returns coming back from that process. Explicitly he are use some topics from economics (model of duopoly, monopoly, cycles), but on the other hand he maintain that very similar approaches is usable in ecology (Lotka-Volterra relations), ethology, psychology, etc., but in this paper the author explores only a few snapshots of familiar economic cases reached by experimentation in virtual economic laboratories created in such software as STELLA and iDmc, and because of short place only for visual impressions and methodical imprints.

Further important purpose of this paper is to distribute out of authors and his fellow colleagues opinions with using virtual laboratories for better and deeper understanding complex phenomena in various learning disciplines. The author uses phenomenological and/or qualitative approaches to meant subjects for elaborating preparatory basis needed for creation mathematical and/or simulation models of authentic learning subject in economy and pointed that the same can be done in systems of biology, ecology, economy and other ones. In this paper he assigns only few possible explicit examples of virtual laboratories, and proclaims that such experimenting in them and shows graphical snapshots of returns coming back from that process are equally possible. The paper is based on more then 20 years experiences of author and his team with using virtual laboratories in education and research.

2. Complexity of real economy: the need of experimentation with complex non-linear model of economic dynamics in virtual economic laboratories

It is true, that real economy; however on first sight, have complex and non-linear discrete dynamic character. So their model must to be non-linear discrete dynamic too. In this sense, some non-linear economic systems are very interesting for economist but on the other hand are extremely difficult for exact analyses. It is in the same way truth that exact, closed-form solution is rarely available or more really spoken such solutions are nil at all. There is no other way than to adopt another strategy, for instance to combine analytical, numerical and graphical methods, or in other words to use advanced methods of experimentation in virtual laboratories built by assistance of computational intelligence. In several cases there arising certain need to study of orbits of non-linear discrete dynamics systems an to give special attention on emerging invariant sets, such as fixed points, periodic, and/or quasi-periodic, more over (deterministic) chaotic sets as we claims in introduction. We can admit that the properties of those sets can be investigated from a professional geometric-topological point of view but such approaches are too difficult for current economist and naturally first of all for students. This is the situation where experimentation in virtual economic laboratories can help in understanding such confused processes going on in socio-economic evolution.

Non-linear discrete dynamic models are useful to explain very irregular, large-amplitude, fluctuations that appear in complex economic and financial systems such as cyclical economic growth, monopoly, duopoly, oligopoly, stock prices and exchange rates. On the other side there are emerging several mathematical difficulties with their analyses. So not every economist and/or students can working with such model in current natural way. Goodwin, Hicks, Kalecki, Kaldor, and Samuelson are among those economist (that is the first post-Keynesian economists) introducing non-linear dynamic models with locally unstable steady states and stable limit cycles to account for the persistence of business cycles, but after further evolution it emerges that problems is more deep and brings globally stable-unstable phenomena. Surprisingly the duopoly model seeming on, at first sight as very simply, may generate very confused and strange dynamical behaviour too. Inspired by the rapid development of deterministic chaos in mathematics and physics, the development of the theory of discrete noninvertible dynamical systems, there is a renewed interest in non-linear endogenous business cycle models in last three decades up to our day. The other interesting point is that at present there are a lot of possibilities in using advanced product of software creators that helps in construction of virtual laboratories in common PC and/or in notebooks. Fortunately, there are several software’s usable for creation virtual economic laboratories, currently. Experimentation in virtual economic laboratories helps in better understanding of complex behaviour in dynamical economic systems. Visualisation of the evolution helps us to see what kind of behaviour is realised in that one or another model evolution.

As a first step of description we begin with virtual laboratory (built in iDmc) experimentation with duopoly model. Subsequently we are exposing laboratory built in STELLA. The same we do with the model of monopolist’s searching and with model of cyclical growth. Unfortunately for understanding following demonstration it is needed some preliminary skill from common economic disciplines. The parallel need is the skill from mathematics at usual university level education requirements. So the author apologize himself for this.
2.1. Experimentation in virtual laboratory created in iDmc with qualitative duopoly model

The duopoly model originated by famous French mathematical economist A. A. Cournot is very appropriate example for demonstration of our experiences with experimentation in virtual laboratories. In spite of seemingly simple nature of that model his behaviour may be surprisingly very complex, namely if we capture into account some learning approaches and/or adaptively behaviour of two players. For explanation of this occurrence of learning players in duopoly model we shall use either their adaptation on one step backward information, or the case without adaptation. The map of duopoly model with adaptation of player has the subsequent mathematical forms

\[ T: \begin{cases} \dot{x} = (1 - \lambda)x + \mu \left( \frac{y}{c_1} - y \right), \\ \dot{y} = (1 - \mu)y + \mu \left( \frac{x}{c_1} - x \right). \end{cases} \]  

\[ M: \begin{cases} x' = x + \lambda \left( \frac{y}{k} - x - y \right), \\ y' = y + \mu \left( \frac{x}{c_1} - x - y \right), \end{cases} \]  

And by shifted coordinates into the centre (into the fixed point)

\[ M: \begin{cases} x' = x + \lambda \left( \frac{y}{k} - x - y \right), \\ y' = y + \mu \left( \sqrt{x - x - y} \right), \end{cases} \]

and in those base we can create virtual laboratory in iDmc for these purposes.

The first snapshots (in Fig. 1 and Fig. 3.), we were made in Basin of attraction routine of iDmc. That case is modification of original Cournot model. It was made by adding an invention about adaptation of players (parameters of adjustment and in above formulas). The snapshot (in Fig. 2.) is made in STELLA. Subsequent snapshots were created without such adaptive parameters.

Fig. 2. Slow approaching to one of nine saddles of Fig. 1. (Made in STELLA for comparison of tools possibilities)

Fig. 3. Fixed point, saddles and twin focuses rotating in opposite directions overlapping basin of attractor

Fig. 4. Double bifurcation map (two parameters basin of attraction) of Cournot duopoly model with adaptive players: “Arnold’s tongues” – red area showing equilibrium and the painted tongues showing the numbers of periodical points of the orbit

Fig. 1. Basin of attraction of Duopoly with adaptation of players

Studying these complex graphical results of simulation in virtual laboratory it must be admitted that before using result of computational intelligence for such activities in learning process anybody learning subjects can imagine anything about deep qualitative processes in similar systems as dynamical duopoly, (triopoly and oligopoly) and so on are. We put our thought that usefulness of such procedures using for better understandings the complex economic problems is, and we are standing for, based on in our opinion that these experiences are indisputable.
2.2. Experimentation with monopolist searching model with linear demand function

Again, only for illustration of possibilities of dealing with introductory and/or first step computational intelligence products in economics we are exhibiting another interesting economic model: that is monopolist (because he is manufacturer of very new product) searching for optimal price and amount of his goods because of lack of market signalling. In the subsequent pictures there are some snapshots from experimentation in virtual laboratory we developed in Excel and iDmc on the base linear demand function and quadratic marginal cost function of manufacturer. For this first demonstration of monopolist’ searching we are using subsequent simple task appropriate for description monopolist possible behaviour by entering future market with new ICT product. We are used following assumptions.

Demand curve function is: \( P_d = 5 - 2Q \). Revenue curve function is: \( R = 5Q - 2Q^2 \). Cost curve function is: \( C = 2Q - Q^2 + 0.7Q^3 \). Profit function is: \( \Pi(Q) = 3Q - Q^2 - 0.7Q^3 \), because in economics is: \( \Pi(Q) = R - C = 5Q - 2Q^2 - 2Q + Q^2 - 0.7Q^3 = 3Q - Q^2 - 0.7Q^3 \). (3)

And after derivation we have: \( \Pi'(Q) = 3 - 2Q - 2.1Q^2 \), and for simulation we shall dealing with quadratic equation: \( 3-2x-2.1x^2=0 \), which after solution given plot of quadratic parabola,

\[
Q = \frac{y + \sqrt{y^2 - 3.62}}{2},
\]

\[
Q = \frac{y - \sqrt{y^2 - 3.62}}{2}
\]

Marginal revenue function is: \( MR = 5 - 4Q \). Marginal cost function is: \( MC = 2 - 2Q + 2.1Q^2 \). The relation of total profit (3) we are uses for construction of searching algorithms. That is based on assumption that monopolist has two real profit value data from market experience, say \( x \) and \( y \). Data \( x \) is the past and \( y \) the present experience. The algorithm for monopolist’s decision on future possible profit is:

\[
z = y + \Delta(y) = y + w \frac{\Pi(x) - \Pi(y)}{y - x},
\]

where \( w \) ... is the longitude of fixed searching step in recursion. Unfortunately in denominator of (4) fraction may happen that \( x=y \), so there is a menace of deterrence division by zero. To evade that problem we are simplifying the job to 2-dim map with general parameters,

\[
T: \begin{cases} x' = y \\ y' = y + w((a - e) - (b - f)(x + y) - g(x^2 + 2xy + y^2)) \end{cases}
\]

using relation (4).

In this base we are created in iDmc device virtual laboratory and by simulation in it we gained subsequent graphs in Trajectory routine in that one.
that one becoming because of such perturbation topologically shifted – the new state isn’t topologically identical with former one. Such processes based on entering the cluster of new ICT and information and knowledge engineering products and new services united with them are drifting all world economy to the shape of global knowledge economy, moreover to global knowledge society and/or to noosphere.

Fig. 9. The trajectories tracking in x-y plane

It is interesting too, that model with linear demand function is more “virulent” in the sense of emerging structural unstable situations. It is important, that some results one can reach also in Excel. Specifically, in spreadsheet we can formulate to same model as 1-dimensional one. That is we can create the model in one column and/or one row sequence as we shall show later.

Fig. 10. Three basic curves and the area of producers interest – the marginal revenue exceed the marginal costs

Fig. 11. Experiment in Excel: the system is attracted by stable focus

In particular cases, for example in the case of new ICT product be able to emerging extraordinary phenomena when rising in prices is parallels with rising the demanded product quantity, something like Giffen’s paradox. Such situation may be coping by right branch of quadratic

parabola. Moreover in other cases after such twin rising can succeed diminishing of demanded product amount again. This three form behaviour we can describes by cubic parabola, or by function of $\tan^{-1}$.

Fig. 12. The evolution (same data as before) in variable space $x\times x$, (or in $y\times y$ if reversibility is warranted)

Fig. 13. Evolution to 4-period orbit

Fig. 14. Rotation in 4-period orbit; rotation number is $\rho=1/4$
process with those parameters can be defined by difference map
\[ x_{n+1} = f(x_n, y_n), \]
\[ y_{n+1} = g(x_n, y_n), \]
where
\[ f(x, y) = y, \]
and
\[ g(x, y) = y + \delta \left( 3.6 - 2.4(x + y) + 0.6(x^3 + xy + y^3) - 0.05(x^3 + x^2y + xy^2 + y^3) \right). \]

2.3. Experimentation with monopolist searching model taken from T. Puu

More interesting case than other is Tõnu Puu’s model of monopolist searching with cubic demand function. One can see partly attractors and partly critical lines in absorption area. Originally T. Puu [12] has been chosen in his model next parameters for searching: \( A = 5.6; B = 2.7; C = 0.62; D = 0.05; E = 2; F = 0.3 \) and \( G = 0.02 \). Searching
2.4. Experimentation with Kaldorian model of cyclical growth (Evolution of model of cyclical growth built on Kaldorian theory base)

For another illustration of CI possibilities in dealing with qualitative economic systems we are exhibiting behaviour of Kaldor like model of cyclical growth as thirds from those specimens that bring difficulties in cognitive processes for students, in this subsection. The virtual laboratory we are created again in iDmc for saving because of bounded area for authors by conference authorities. We are directly decrypting of laboratory in LUA-JAVA language is as follows:

```
--@@
name = "Kaldorian model"
description = "See Model references"
type = "D"
parameters = {"b", "c", "d", "Y", "K"}
variables = {"Y", "K"}
function f(a, b, c, d, Y, K)
    Y1 = (1-b-a)*Y+math.atan(K)
    K1 = -c*b*Y+(1-c*d)*K+c*math.atan(K)
    return Y1, K1
end
function Jf(a, b, c, d, Y, K)
    return 1-b-a, 1/(1 + K^2),
         -c*b, 1-c*d+c/(1+K^2)
end
```

We are used model formalism motivated from (Agliari et al., 2007 [2]), that is we are using their two-dimensional nonlinear discrete-time dynamic system (as topological map):

\[
M :\begin{cases}
    Y_{n+1} = Y_n + \mu [\alpha Y_n + \gamma \left( \frac{\sigma}{\delta} - K_n \right) + \arctan (Y_n - \omega) - \sigma Y_n] \\
    K_{n+1} = (1 - \delta)K_n + \sigma Y_n + \gamma \left( \frac{\sigma}{\delta} - K_n \right) + \arctan (Y_n - \omega)
\end{cases}
\]

(8)

Fig. 24. Snapshot of multitude of critical curves in absorption area: model on Kaldorian base

Fig. 25. The multitude of critical curves presenting attractive and repulsive evolving in Kaldorian model behaviour
3. Conclusions

With this paper the author would like to advocate for wider using of advanced products of computational intelligence for investigation in complex economic phenomena appropriate for building virtual laboratories and for building virtual co-developers living in appropriate software useful to researchers, teachers and students in economic science. He proposes the idea of integrative approaches aided by product of advanced CI in this paper. Namely because only integrative methods aided by sophisticated CI products is able completely deal with new, unprecedented process emerging in contemporary global knowledge society. In the virtual laboratory simulation of dynamical behaviour of economic systems with nature of sub-critical Neimark-Sacker bifurcation is very important for understanding hidden complexities in them. In such virtual experiments that is to say, the emergence and existence of a repelling invariant closed curve which bounds the basin of attraction of the stable fixed point implies for economic scholars fundamentally important disposition the very process. Those is lying in fact that small perturbations of the system have no effects on its dynamical behaviour, while large enough disturb may drift the system to another attractor. In this sense the system after small perturbation keeping on structurally stable state forever. By other word the system after such perturbation is keeping on topologically equivalent state and/or dynamical regime. In such situation upon the whole system the basic fixed point governs. We are exhibited above that those qualitative regimes realizing such motion is either stable node, and/or stable focus depending on the longevity size of probing step parameter.

But the situation, first of all for conventional (main stream) economist’s buildings their theories, is not as optimistic as it like in first sight. Unfortunately, the size of disturbances, even sometimes there are in clusters, emerging in real economies are outlying from one that we may called “small” as we do above. In such situation our
attractive fixed point losing his attractive force and delivering it to closed invariant curve. But descriptive situation is a little more complex. For example, if the shock is so large enough that may drift the system to another attractor the scholar have to reveal hidden coherences, such as requirements of the coexistence of the fixed point with a different attracting set are.

In this paper we are done some effort to exhibit how experimentation in virtual economic laboratories can help in gaining better, more effective and efficient, more perpetual knowledge of complex behaviour in non-linear economic systems, but it is right that only in abridged, fragmentary and forced scrapped range. So we offer relatively simple bundle of devices available for theorizing purposes in economics at least for preliminary phase of investigation and theory building. We based those exhibitions on several years experiences achieved in education in Slovak university of Technology Bratislava. That task is difficult not only for very contents of this branch of economics and decision making process’s but in first plane for the truth that exhibit in written on paper form those achievements of possibility directly watching of running experiments is basically impossible. So we must limit our effort only for exhibition of finished product of experimentation in static graphical form such as trajectories, bifurcation maps, and basins of attractors, manifolds, cobwebs animations, critical curves and attractors in absorbing areas and so on. In such circumstances we focused attention only on registration the facts that such devices, tool and methods is available and everybody can use it on cognitive processes in minimal claims for former special mathematical education. By name we focusing attention in one hand on general aspects of the object without concretization of inherent content of tasks, and on the other hand we are reveal proper economic content of three particular economic theory and/or model: first - monopoly, second - duopoly and third - cyclical growth, unfortunately only in inequitable non continuous part inputs, because of shortage of places in this proceedings.

In circumstance of monopoly and monopolist we can, at least by way of joke, said that we have for them two messages: one good message and the other bad one. The good SMS-message is that he/her can independently set the price and amount of their product for market. The bad e-mail is that he/her doesn’t know how and what level and/or volume have to set down. As T. Puu noticed in his famous book (T. Puu, 1999, p.130) it may be a bit hard for economists, nourish with textbook monopoly theory, to digest that the monopolists does not know all about the market and may even behave in a way seeming erratic. He admits that it can be prohibitively expensive to get all the information the monopolistic firm needs, and that the knowledge acquired may be only local. We can, pragmatically think, parallel with T. Puu, that monopolist might have no idea, at all, of the true global outline of the marginal revenue curve (see for example Fig. 9-16), as to how many humps there are and where they are located, etc.

It is known that economics and economists recognize two opposite market forms: monopoly and perfect competition. We can, similarly at least as a joke, say that competitive market is wiser than monopolist. Such
markets “know” the appropriate price and amount (because of unseen hand of market is still in work). But beside these two opposite market form there is third form too: the duopoly and/or oligopoly ones. The problem is, that duopoly, though contextually the first step from monopoly towards perfect competition is analytically not a case of intermediate complexity, but more complicated that before mentioned extremes (see Puu [12 ], 1999, p.133-158). From this point of view A. Cournot was a natural first founder of the theory of iterated two person game. Simulation in virtual laboratory helps in understanding that complicated process too.

In third case we have considered a particular version of the Kaldor business cycle model in discrete-time, which has been proposed in group of Italian economist-mathematicians Agliari et al. (See Agliari et al., 2007). The focus of that part of paper has been focused to phenomena of coexistence of attractors attracting and repelling invariant curves which are associated with phenomena of coexistence of attractors and intricate structures of the basins of attraction. This has been illustrated within the well known nonlinear framework of the Kaldor model. It has partly shown, based also on the excellent works of “Italian group” that bifurcation sequences involve homoclinic (or heteroclinical) tangles (like general approach visualised in Fig. 17-23), associated with saddle points or saddle cycles of different period, and that the “intermediate” scenarios are of necessity accompanied by complex structures of the basins of attraction. This strengthens the role of homoclinic bifurcation theory (which has been recently introduced in economics in order to explain the routes of complex attractors which are often observed in numerical experiments) as a tool of analysis of global dynamic phenomena.

Finally we would like to thank all members of, as we attempt named them, “Italian group” for their excellent achievements in named branch of economics and, first of all, warm thank is belong to professors Anna Agliari, Laura Gardini and Marji Lines for helps with solving several problems of these complex topics in the learning processes at Slovak University of Technology Bratislava.

4. References